18.01

Single Variable Calculus

SINGLE VARIABLE CALCULUS

Differentiation

Lecture 3: Limits and continuity

Definition. Let f be defined near a. We say that the limit of f(x) as x approaches a equals L, written lim f(x) = L, when f(x) gets arbitrarily close to L for all x sufficiently close to a.

Definition. f is continuous at a when lim f(x) = f(a) as x approaches a.

Informally: small changes in the input produce small changes in the output.

Theorem (squeeze). If g(x) <= f(x) <= h(x) near a and g and h share the limit L at a, then f also has limit L at a.

Example. lim (sin x)/x = 1 as x approaches 0, by the squeeze theorem applied to cos x <= (sin x)/x <= 1.

Lecture 4: The derivative

Definition. The derivative of f at a is $f'(a) = \lim (f(a+h) - f(a)) / h$
as h approaches 0, when this limit exists.

Every differentiable function is continuous; the converse fails, as the absolute value function shows at 0.

Power rule. For every integer n, $d/dx\, x^n = n\, x^{(n-1)}$.
Examples. $d/dx\, x^2 = 2x$, $d/dx\, x^3 = 3x^2$, $d/dx\, 1/x = -1/x^2$.

The sum and product rules follow from the definition by computation.

LINK

Lecture videos (MIT OpenCourseWare)

https://ocw.mit.edu/courses/18-01sc-single-variable-calculus-fall-2010/

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